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Development of Mathematical Modelling for Predicting Spray Mass Flux on Tree Canopies

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***Abstract**: This study is to develop mathematical model that could theoretically estimate spray mass flux of spray materials of a selected sprayer at different operating conditions in an orchard farm. Investigation was conducted and data were collected on randomly selected orchard tree of population of (n)100. A paired t-test was conducted and the result revealed that the calculated t- values (0.769) is less than the table value (3.25) at 0.01 level of significance, with (r 2) of 0.98, the Root mean Square Error (RMSE), the standard deviation (S d) 0.059 and 0.0065 respectively. All the inputs variables of the model developed indicate high degree of precision with which the treatment values were compared and are a good index of reliability of the experimental results and the sensitivity of developed predicted models to change in the constituent independent variables is negligible and not significant.*

***Keywords:** spray mass flux, modelling, orchard tree, canopies.*

1. INTRODUCTION

The use of pesticide was widely studied due to the economic and environmental costs. More specialized methods were found for pesticide and plant protection products (PPP). Initially, distribution of agrochemicals was shared flawlessly on the gardens, due to that dose of each unit ground area was expressed as a dose rate. Pesticide application describes the practical way where pesticides, (such as Compounds, fungicides, insecticides, or nematode control representatives) are sent to their own biological goals (e.g., pest organism, harvest or another plant). Public concern with the use of pesticides has emphasized the requirement to make this process as effective as possible, so as to minimize their discharge to the environment and human exposure (such as operators, bystanders, and customers of produce). The custom of pest control by the logical application of pesticides is supremely multi-disciplinary, combining numerous facets of biology and chemistry together: agronomy, technology, meteorology, socioeconomics, and general health, jointly with newer areas like biotechnology and data science. Orchards were identified by a large number of trees in small farms using height, density and a variety of tree shapes. It was led to sprayer manufacturers endorsing the design and development of machines with direct declining of traditional air blast sprayers with easy adjustment to match the crop canopy. An approach to achieving this objective was fitting of adjustable vertical air outlets. The arrangement of these sprayers takes many different aspects, some of which have been studied and others dose not checked. The most calculated solutions for improving spray control and reducing drift losses by adjusting air outlet of the sprayer that matches with canopy of the crop. Taking this approach to the extreme development of air-assisted sprayers, in which shield rows and recycling systems have the full edge of target spray liquid.

Among the most typical kinds of pesticide program, particularly in agriculture that is traditional, is using mechanical sprayers. Sprayers convert a chemical formula, frequently containing a combination of water (or some other liquid compound provider (like fertilizer) and compound, into droplets, which is big rain-type drops or miniature almost- invisible particles. This transformation is achieved by pushing the spray mix under stress by means of a spray nozzle. The dimensions of droplets could be changed through the use of nozzle dimensions, or by changing even a combo of both, or the strain under which it's forced. Droplets have the benefit of being less vulnerable to spray drift but need more water per unit of property. End conditions are demanded, although Because of static electricity droplets can optimize contact with a target organism. Air blast sprayers or air delivery have been utilized to use foliar nutrition, plant growth regulators and pesticides. These substances are applied by them as fluids. Air blast sprayers have alterations in air delivery methods and the fluid which allow tailoring the software to match a range of orchard requirements.

The cost and Efficacy effectiveness of orchard Pest Control The abilities of sprayer operators and supervisors that assess orchard requirements and change operating methods and machine configurations to maximize the performance of sprayers influence Management applications. A mixture of timing, equipment functionality, ability, and compound selection are vital for optimum outcomes. Airflow characteristics that affect coverage Include air volume (CFM: cubic feet per second) and speed (FPM: ft per second). Enthusiast type and rate, size design influences these parameters so on. As previous and such remarks indicate, several variables, most Interactive are included with air shipping sprayer performance. Performance data regarding Several of These variables for sprayers Aren't Generally offered.

All spraying requires care and attention to details to achieve good results, but orchard spraying is generally more challenging and difficult to perform well than boom spraying in fields. Field spraying tends to be a two-dimensional problem involving an area to be covered. Orchard spraying adds a third dimension—height—and concern for the volume in the target area (i.e., the size of trees). [1] Other significant differences are the much greater distances between nozzles and target in orchard spraying and the amount of air used to carry spray to the tree. All spraying requires care and attention to details to achieve good results, but orchard spraying is generally more challenging and difficult to perform well than boom spraying in fields. Field spraying tends to be a two-dimensional problem involving an area to be covered. Orchard spraying adds a third dimension—height—and concern for the volume in the target area (i.e., the size of trees). Other significant differences are the much greater distances between nozzles and target in orchard spraying and the amount of air used to carry spray to the tree.

[2] reported also that for sometimes, many researchers were analyzed and developed dose models for orchards, groves and vineyards. The purpose of pesticides needs to be technically effective. It includes more than the specific application of the selected mass of spray mixture. Since the cost of chemicals and applications has increased, here need more efficiency in spraying has become necessary. Recent trends in air-assisted spray applications are based on the use of non-conventional sprayers to increase in treatment options and reduce pollution. Poor match was appeared between the conventional air blast sprayer and the plant geometry in losses of insecticide.

Research activities of the project are to ascertain system operating parameters and transportation systems impacting supply uniformity for air. Deposition diminished with depth in mango tree canopies, and the rate of reduction was influenced by spray volume and sprayer airflow speed [3]. Some properties like tree construction flux, droplet size, spray volume rate speed, wind speed, air flow speed condition, temperature, and humidity are essential from the processes. It was discovered that these factors influence spray transportation to and inside. For harvest applications employing standard sprayers, spray sediment in the plant canopy is dependent upon droplet size, droplet speed, spray volume rate [4], wind speed, and tree construction). Droplet size is dependent upon atomizer form, nozzle pressure/atomizer rate, liquid flow rate, liquid components [5], and atmospheric temperature and relative humidity. Whilst Crop structure could be characterized by the canopy shape, volume, and density speed is a function of droplet size, wind speed, sprayer floor speed, and air flow velocity. [6] conducted a study to find out the effect of different parameter on spraying. The operating speed of sprayer showed significant effect on spray droplets deposition, distribution by conspicuously greater at lower operating speed. In air assisted sprayer, increasing blower speed significantly increased drift force and increased tree canopy area in spraying. The efficiency of spray deposition increased with increase in blower speed. The conveying efficiency of spray particles, penetration and distribution of pesticide were increased with increase in operating pressure. The performance of sprayer was found better with increase in operating pressure (1 to 15 kg/cm^2) of spraying.

[7] developed a mathematical model that could predict the total mass flux. In his studies he assumed that sprays resemble jets, as both expand and decelerate after exit from the orifice and entrain ambient fluid.
He observed that the spray mass flux decreases with increasing distance from the sprayer due to air entrainment at spray jet boundaries, droplet settlement, and decrease in spray velocity. An assumption was made that the liquid mass evaporated from the spray is entirely absorbed by the air in the control volume. The spray mass flux at distance x becomes: The total mass flux at the sprayer air outlet can be estimated as [8]:

$$\mathrm{M}_x = \frac{\rho_a Q_{ax} + \rho_l Q_{lx}}{A_x} \quad \text{..........} \quad (1)$$

where
M_x = mass flux at distance x, kg/m^{2}s
A_x = the cross-sectional area of the spray jet at distance x, m
Q_{lx} = the liquid flow rate at distance x. at distance x, m^3/s
Q_{ao} = the airflow rate at distance x. m^3/s
ρ_l = density of liquid at distance x, kg/m^3
ρ_a = density of air at distance x, kg/m^3

He estimated the mass flux (M_x), by calculating the cross-sectional area of the spray jet (A_x), the liquid flow rate (Q_{lx}), and the airflow rate (Q_{ax}) at distance x.

The broad objective of this study is to develop mathematical models that could theoretically estimate spray mass flux of spray materials of a selected sprayer at different operating conditions in an orchard farm. The models will enable researchers to organize their theoretical belief and empirical observations about the spray mass flux system and to deduce the logical implications of the organization.

2. MATERIALS AND METHOD

The test materials used were a motorized mist blower a manometer, an anemometer, a tachometer, a measuring tape, photographic papers or tracers, a stop watch and trees of various sizes in the orchard farm. The sprayer was calibrated in the crop protection machinery laboratory of the Agricultural Engineering Department, Ahmadu Bello University, Zaria, Nigeria. The Sprayer used for this study was a tuber super k90 motorized knapsack mist blower. The test trees are Mango tree of the same variety and different heights and located at an orchard farm of the Faculty of Agricultural Sciences, National Open University of Nigeria, kilometer 4, Kaduna-Zaria expressway, Rigachikun, Kaduna State.

2.1 Theoretical Considerations: Model Development

Mathematical model is a mathematical representation of the conceptual view of a process based on physical theory, empirical observations or a combination of both. In developing a mathematical prediction model, some important steps were considered for proper formulation. The steps consist of model formulation and verification. The prediction of spray mass flux was done considering the change in mass flux and velocity of the spray as it moves toward the canopy. The assumptions made in this modeling are as follows;

1. the sprayer's air stream is treated as a plane symmetric jet in this study because the sprayer has air outlets that generate fan jets which are amenable to plane jet treatment [9].
2. the liquid mass evaporated from the spray is entirely absorbed by the air in the control volume.
3. in determining airflow rate, there is the absence of co-flow (no ambient air parallel to jet air)
4. the change in distance χ, ($\Delta\chi$) is so small that there is no change in air velocity while moving through the distance.
5. the air jet adjusts during travel from the sprayer air outlet to the tree canopy.

2.2 Determination of Total Mass Flux Using Dimensional Analysis

The total mass flux (M) at the sprayer air outlet was determined using dimensionless analysis by relating the total mass flux (M) to the factors influencing it. Assumed that the variables of importance are the cross-sectional area of the spray jet, the liquid flow rate, airflow rate, spray air density and distance between sprayer outlet and canopy boundaries.
The use of dimensional analysis to form a prediction equation involved the following steps:

1) Establishment of the physical variables, the variables to be used in this analysis are presented below;

$$M = f_m (X, Q_a, Q_l, A, \rho_a.) \quad \ldots\ldots (2)$$

$$f_m = (M, X, Q_a Q_l, A, \rho_a.)$$

where,

M	=	spray mass flux, kg/m^2
Q_a	=	airflow rate, m^3/s
Q_l	=	liquid flow rate, m^3/s
ρ_a	=	density of air.kg/m^3
X	=	dist. between sprayer outlet and canopy boundaries, m
A	=	cross sectional area of spray jet, m
f_m	=	functional notation

2) Selection of the basic dimensions such as mass, length and time represented as M, L and T respectively, and the tabulation of the chosen pertinent variable quantities with their corresponding dimensions are shown in Table 1,
3) Formulation of dimensionless groups called π – terms using the matrix method, while the number of these groups was furnished by Buckingham pi Theorem. The Buckingham pi theorem states that the number of dimensionless and independent quantities required to express a relationship among variables in any phenomenon is equal to the number of quantities involved minus the number of dimensions in which those quantities may be measured. Expressing this mathematically;

$$S = N - B \quad \ldots\ldots (3)$$

where

S = the number of π – terms,
N = the total number of quantities involved,
B = the number of basic dimensions involved.

The number of π – terms required to form the spray mass flux; equation have been determined. Thus, the equations in the form of π – terms are given as:

$$\pi_1 = F_m(\pi_2, \pi_3) \quad \ldots\ldots (4)$$

where,

F_m = spray mass flux functional notation

From the solutions, the set of π- term corresponding to the above equation for the spray mass flux, was determined as presented in the equation below;

$$\frac{MA}{Q_a \rho_a} = F\left(\frac{X}{A^{\frac{1}{2}}}, \frac{Q_l}{Q_a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

Table 1: Variables and dimension spray mass flux.

Variables	Symbol	Unit	Dimension [M][L][T]
Mass flux	M	Kg/m^2s	$ML^{-2}T^{-1}$
Distance between sprayer outlet and canopy boundaries	X	M	L
Liquid flow rate	Q_l	m^3/s	L^3T^{-1}
Airflow rate	Q_a	m^3/s	L^3T^{-1}
Cross sectional area	A	m^2	L^2
Spray air density	ρ_a	Kg/m^3	ML^{-3}

2.3 Spray Mass Flux Prediction Equations

The formation of prediction equations for spray mass flux, involved the determination of the function for the general equation. This entailed the determination of component equations, the determination of the mode of combination to form the prediction equation, the determination of the value of the constant term for the mode of the combination, and the formation of the general prediction equation.

2.3.1 Determination of the component equations

The component equations were formed from the experimental data on the pi- terms. They represented the regression equations of the $\pi - terms$ data which are presented in Table 1for spray mass flux and consists of data of the three $\pi - terms$ required for the determination of the prediction equations. The spray mass flux component equation consists of two equations that were formed from the plots of π_{1m} *against* π_{2m} *and* π_{1m} *against* π_{3m} from table 2. The graph is shown in Figures 1 and 2. In each of the figures, two linear line graphs are presented, one of which is plot of a supplementary data (as explained earlier) to test the conformity of the whole data.

Table 2: The spray mass flux pi terms data

$\pi_{1m} = MA/ Q_a\rho_a$		$\pi_{2m} = X/ A^{1/2}$		$\pi_{3m} = Q_l/ Q_a$	
16.66	21.05	0.081	0.15	0.002	0.02
17.70	16.41	0.19	0.26	0.017	0.015
18.66	17.18	0.34	0.38	0.018	0.016
16.49	15.10	0.42	0.50	0.016	0.015
15.20	16.49	0.51	0.59	0.014	0.016
15.24	17.09	0.62	0.70	0.014	0.016
13.01	13.95	0.72	0.80	0.012	0.013
12.40	13.19	0.83	0.91	0.011	0.012
12.67	13.52	0.95	1.02	0.011	0.013
12.04	13.07	1.01	1.09	0.011	0.012

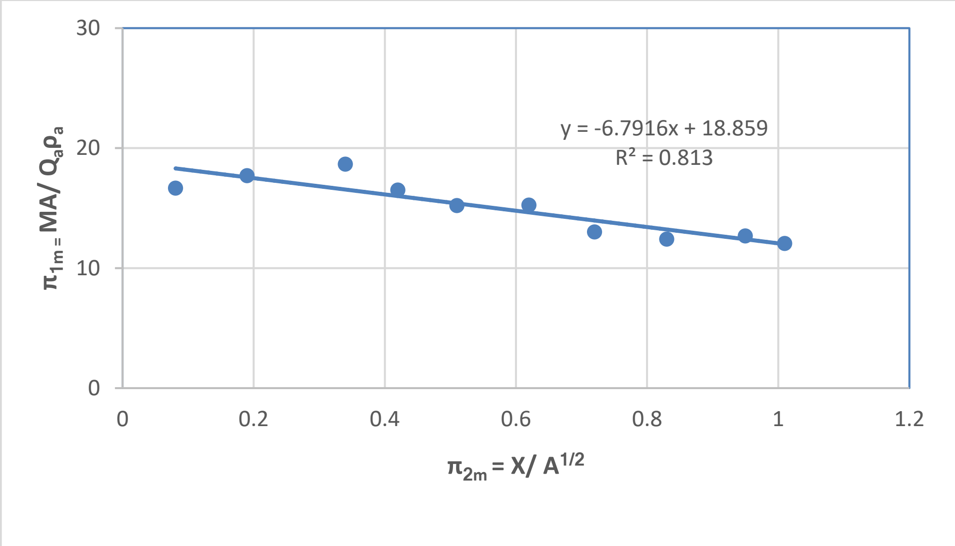

Figure 3.2a: Plot of Π_{1m} vs Π_{2m} for original data

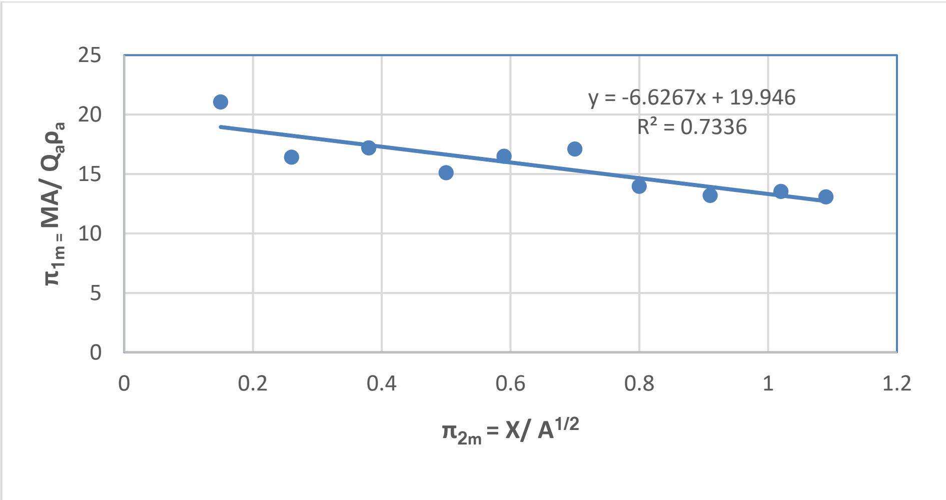

Figure 3.2b: plot of Π_{1m} vs Π_{2m} for supplementary data

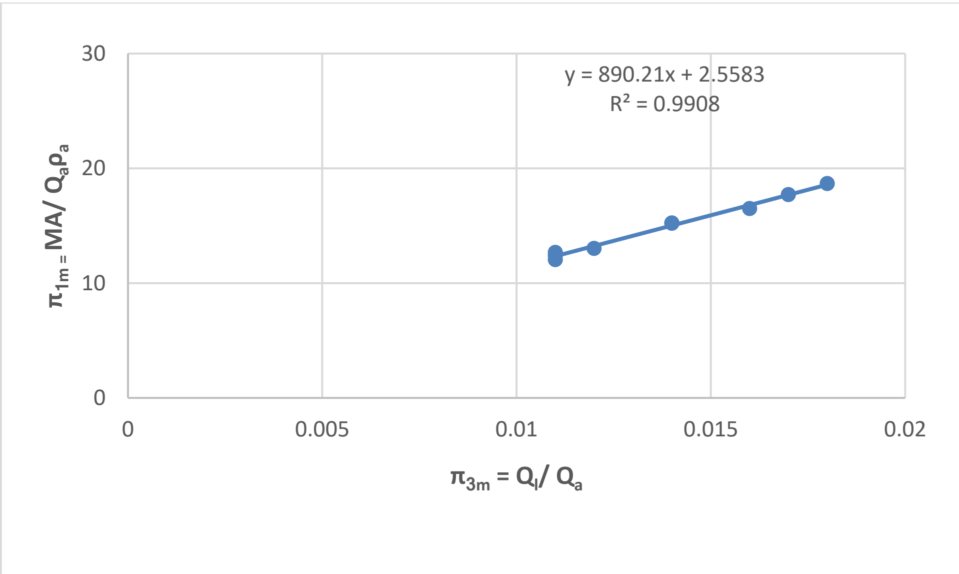

Figure 3.3a: Plot Π_{1m} vs Π_{3m} for original data

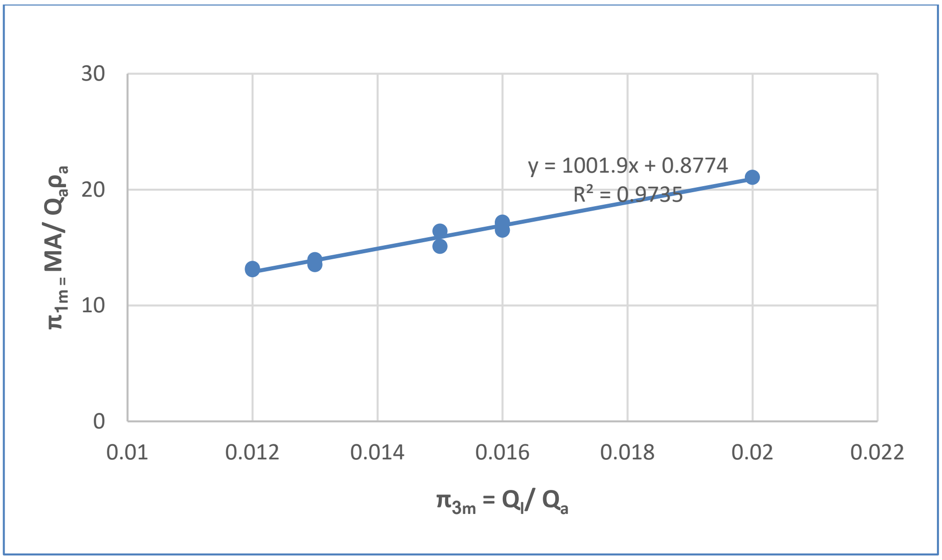

Figure 3.3b: Plot Π_{1m} vs Π_{3m} for supplementary Data

The regression analysis gave coefficients of determination 0.81 and 0.97 and 0.73 and 0.99 for original and supplementary data for the regression equations representing the component equations given as:

$\pi_{1m} = -6.7916\pi_{2m} + 18.859$.. (5a)
$\pi_{1m} = -6.6267\pi_{2m} + 19.946$.. (5b)
$\pi_{1m} = 890.21\pi_{2m} + 2.5583$...(6a)
$\pi_{1m} = 1001.9\pi_{3m} + 0.8774$...(6b)

2.3.2 The Component Equations Combination

The component equation was combined by summation to form the general prediction equation. The selection satisfied the criteria as explained by [10]. The form of component equations gave an insight on the likely condition of mode of combination would satisfy. [10] explained that: When a set of component equations plot to form a plane surface on space (linear space), such equations would be valid for combination as summation and will have the form of $y = a + bx$.

2.3.3 Validity of Combining the Component Equation by Summation for spray mass flux equation.

[10] stated that for valid combination of component equations by summation, the right-hand side of equation must be equal to the left-hand side. When the number of π -terms involved are 3, the equation becomes:

$$f(\pi_2, \bar{\pi}_3) - f(\bar{\pi}_2, \bar{\pi}_3) = f(\pi_2, \bar{\bar{\pi}}_3) - f(\bar{\pi}_2, \bar{\bar{\pi}}_3) \qquad (7)$$

where, $\bar{\pi}_3$ and $\bar{\pi}_3$ constants

$\bar{\bar{\pi}}_3$ = a constant value of π_3 at a supplementary data.

Considering the terms in equation above only one of the component equations with its supplementary equation is involved in the test. The specific tests for summing component equation for spray mass flux are conducted as presented below:
One of the component equations for the spray mass flux is given as:
$\pi_{1m} = -6.916\pi_{2m} + 18.59 = f_{1m}\{\pi_2|\pi_3\}$
Taken $\bar{\pi}_{2m}$ as the average sum of π,

$\pi_{2m} = 0.567$

$f_{1m}(\bar{\pi}_2, \bar{\pi}_3) = -6.7916(0.567) + 18.59$
$= 14.74$

A supplementary equation when π_3 is held at another value $\bar{\pi}_3$ is:

$\pi_{1m} = -6.6267\pi_{2m} + 19.946... = f_{1m}\{\pi_2|\pi_3\}$

$\bar{\pi}_{2m} = 0.79$

$f_{1m(}(\bar{\pi}_2, \bar{\bar{\pi}}_3) = -6.6267(0.79) + 19.946 = 14.70$

Substituting the component and supplementary equations and their constant values in equation.

$\pi_{1m} = -6.7916\pi_{2m} + 18.859$ -14.74 = $\pi_{1m} = -6.6267\pi_{2m} + 19.946$ – 14.7
$-6.7916\pi_{2m} + 18.859$ -14.74 = $-6.6267\pi_{2m} + 19.946$ – 14.7
$-6.7916\pi_{2m} + 4.119 = \pi_{1m} = -6.6267\pi_{2m} + 5.246$ … … … … … … … … … … … … (8)

Since the right-hand side is approximately equal to the left-hand side, this means that the combination by summation of spray mass flux component equation is valid.

2.3.4 Determination of the constant of summation

The constant of summation for S number of $\pi - terms$ is given as:

$$C = (s-2) f(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4, ...\bar{\pi}_s) \quad \text{……………………………………… (9)}$$

where,
C = Constant of summation, S = Number of π -terms and F = Functional notation
The constant term expression when the number of π -terms is 3, becomes:

$C = f_1(\bar{\pi}_2, \bar{\pi}_3), or \ C = f_2(\bar{\pi}_2, \bar{\pi}_3)$. The constant can be evaluated from any of the component equations as each should give the same value as the other [9]. The components equation for spray mass flux equation is given and the constant is evaluated as follows;

$C_m = f_{1m}\ (\bar{\pi}_2, \bar{\pi}_3) - 6.7916\pi_{2m} + 18.859$

$\bar{\pi}_2$ and $\bar{\pi}_3$ from the table (3.4) are 0.567 and 0.64

$C_m \ = -6.791_{x2m} + 18.859$

$\therefore$ C_m = 15.001 or

$\pi_{1m} \ = \ 890.21\pi_{3m} + 2.558$

$\therefore$ C_m = 15.701

The constant calculated from either of the component equations are approximately equal. Thus, the constant terms can be determined from any of the component equations.

2.3.5 Formation of General Prediction Equation

The general prediction equation for a system or process involving $S\pi$ -terms formed by addition of the component equations given as:

$$\pi_1 = f_1(\pi_2, \bar{\pi}_3, \bar{\pi}_4, \ldots \bar{\pi}_s) + f_2(\bar{\pi}_2, \pi_3, \bar{\pi}_4, \ldots \bar{\pi}_s) + f_3(\bar{\pi}_2, \bar{\pi}_3, \pi_4, \ldots \bar{\pi}_s) + f_s(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4, \ldots \bar{\pi}_s) - C$$

……………… (10)

where C is constant terms of summation. The prediction equation for spray mass flux, spray air velocity and spray deposition involve $3\pi - terms$ each. Thus, the above equation when reduced to three $\pi - terms$ becomes:

$$\pi_1 = f_1(\pi_2, \bar{\pi}_3) + f_2(\bar{\pi}_2, \pi_3) - C \quad \text{……………………………………….. (11)}$$

The general equation for spray mass flux, spray air velocity and spray deposition are obtained by summing the respective component equations. The relationship between the functions and π_1 are

$$\pi_{1m} = f_{1m}(\pi_2, \bar{\pi}_3) + f_{2m}(\bar{\pi}_2, \pi_3) - C_m \text{ ………………………………………..(12)}$$

The section above shows the algebraic summation of the component equations and the constant of the summation ((C_m) for spray mass flux. The result obtained is:

$$\pi_{1m} = -6.7534\pi_{2m} + 890.21\pi_{3m} + 6.3872 \text{ ……………………………...13}$$

Substituting for symbolic representation of-terms in the above equation, resulted in equations 1.10 below, which represents the prediction equation for spray mass flux (M).

$$M = \frac{\rho_a}{A}\left[-6.7534XQ_a A^{-0.5} + 890.21Q_l + 6.3872Q_a\right] \text{……………14}$$

2.4 Measurement of Sprayer Parameters

The Width and height Of Sprayer Air Outlet, Air Flow Rate at Outlet (Q_a), Distance Ground Speed and Wind Speed were measured with: A measuring steel ruler (model: Stanley), a pitot tube manometer at 56 nodes, a tape rule (model: Stanley), tachometer (model: Machley) and A wind cup anemometer (model: Smith) respectively. The recorded data are presented table 4.

The average wind speed was 2.89m/s and the wind direction were east. Other factors determined were the Rows in grove (north to south), Spray released direction: east to west and operator's direction: north to south

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2.4.1 Determination of spray mass flux

Spray mass flux: The spray mass flux upon entrance to the canopy was determined using the following procedure.

Procedure:

The sprayer was run at a selected speed of 0.8km/hr. then spraying was done initially on the ground to obtain the swath width at that speed then at the same speed, spraying was done for 20s while the spray was collected in a container and measured to know the amount of spray collected. The test was repeated for various readings of speed and presented in table 4. Subsequently the spray mass flux was determined by calculation after the area of spray was determined and taking into consideration that spray mass flux is a function of amount of spray in kilogram over area at a given time.

Spray jet cross – sectional area (A)

The spray jet width was taken as the width of the air outlet therefore, the spray jet was considered as a plane jet and can be estimated using an expansion coefficient of 0.1m per meter of a distance x. [11]. The cross sectional was determined by multiplying the width of jet (b_x) by the height of tree at distances of x.

2.4.2 Laboratory experiment

The laboratory test was conducted in the Department of Agricultural Engineering, Ahmadu Bello University. Zaria. The main objective of this test was to determine the liquid flow rate of the sprayer in view.

Test procedure

The flow rate test was carried out on the machine using various restrictors and water as a test fluid. The tank was filled with of water and flow rates of high-volume nozzle were determined using restrictors 1 -10 and the results are presented in table 3.

Table 3: Experimental data obtained on sprayer application parameters

Parameters	Airflow rate (m^2/sec)	Liquid flow rate (l/s)	Spray air velocity (m/s)	Spray Jet cross-sectional Area (m^2)	Spray mass flux (kg/m^2s)	Distance between sprayer outlet and tree (m)
Symbol	**Q_a**	**Q_l**	**V**	**A**	**M**	**X**
	0.132	0.0002	15.6	1.01	1.46	0.1
	0.197	0.0033	18.8	1.15	3.04	0.2
	0.237	0.0042	21.2	0.96	4.62	0.3
	0.283	0.0044	24.9	0.90	5.20	0.4
	0.33	0.0047	28.6	0.96	5.24	0.5
	0.35	0.005	32.3	0.94	5.69	0.6
	0.44	0.0053	38.2	0.87	6.62	0.7
	0.49	0.0056	42.4	0.86	7.2	0.8
	0.51	0.006	44	0.93	6.9	0.9
	0.56	0.0062	47.3	0.90	6.8	1.0

3. RESULTS AND DISCUSSION

3.1 Verification of Spray Mass Flux Equation

The spray mass flux model equation is given in equation 14. The computed output of the equation and the measured data are presented in table 4

Table 4: Data for predicted and measured spray mass flux

S/No	Measured (X_m)	Predicted (X_p)
1	1.02	1.05
2	4.00	3.80
3	4.78	4.50
4	5.05	5.07
5	5.10	5.03
6	5.15	5.09
7	5.39	5.25
8	5.20	5.40
9	4.99	5.30
10	5.26	5.50

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The computed output was plotted against the measured as presented in Figure 3. The regression analysis gave the values of the slope and intercept as 0.9672. The coefficient of determination (r^2) was found to be 0.98. Applying the hypothesis b-β =0 and α=α_o, the computed t_b (0.930) and t_a (0.992) were less than the table (1.86) at 8 degrees of freedom at 1% level of significance. Therefore, there are no significance difference between both the predicted and measured slope (b-β =0) and the between α and α_o (that is, the intercept is zero). hence the two tests showed that the predicted model is not significantly different from the 1:1 model equation. A paired t-test was conducted and the result revealed that the calculated t- values (0.769) is less than the table value (3.25) at 0.01 level of significance. This shows that there is no significant difference between the predicted and measured π_1 values for spray mass flux. The Root mean Square Error (RMSE) and the standard deviation ($S_{d)}$ for the variation were found to 0.059 and 0.0065 respectively. The test of the significance of the coefficient of determination (r^2) using t- table revealed that it is highly significant with calculated value of 19.799 which is greater than the value of 3.355 at 0.01 probability levels.

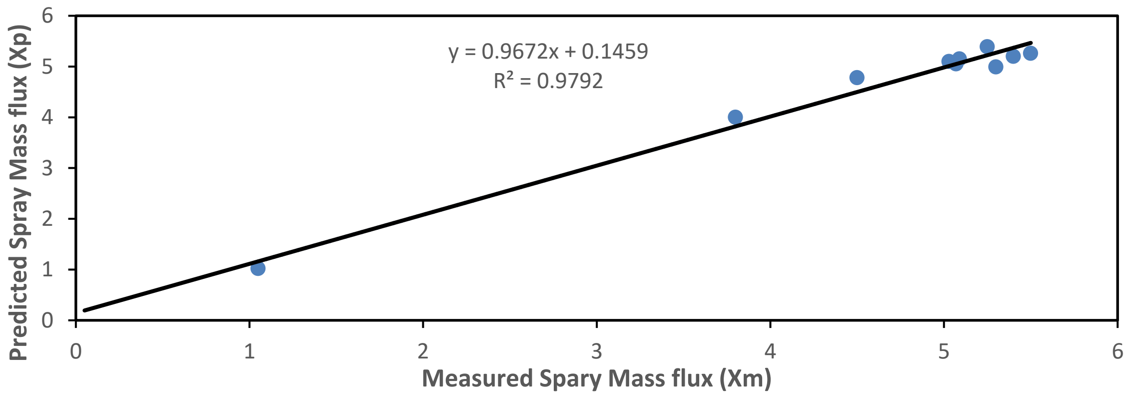

Figure 4: Graph of Plot of Measured Spray Mass Flus Vs Predicted Mass Flux

3.2 Validation of Spray Mass Flux

The sensitivity of measured parameters to Spray Mass Flux model was determined. [11] defined the sensitivity of a model to a given parameter as the rate of change in the output of the model with respect to the change in the value of the parameters while keeping other parameters constant. Sensitivity analysis enabled the relative role of each variable to the developed model to be known. This was done by noting the change of dependent variable in the model with respect to the change in each of the several independent variables.

Sensitivity analysis was done by mathematically differentiating the developed models to derive equation for the rate of change of the independent variable with respect to each dependent variable [12]. [13] showed that sensitivity equation (also called error equations) is developed for a function using:

$$N = f(u_1, u_2, \ldots\ldots u_n) \quad \ldots\ldots (17)$$

where,
N= Dependent Variables
U_i-U_n=Independent Variables
By first writing

$$N + \Delta N = f(u_1 + \Delta u_1, u_2 + \Delta u_2, \ldots\ldots u_n + \Delta u_n) \quad \ldots\ldots (18)$$

Then applying Taylor's theorem and neglecting squares, product and higher powers in the expansion yield:

$$\Delta N = \frac{\partial N}{\partial u_1}\Delta u_1 + \frac{\partial N}{\partial u_2}\Delta u_2 + \frac{\Delta N}{\Delta U_n} + \Delta U_n \ldots\ldots \quad \ldots\ldots (19)$$

Relative changes or Errors was defined as

$$N\xi = \frac{\Delta N}{N} \quad \ldots\ldots 20$$

$$u\xi = \frac{\Delta u}{u} \quad \ldots\ldots 21$$

Substituting equation (5.3) and (5.5) into (5.4), provides the general equation

$$N\xi = \left(\frac{\partial N}{\partial u_1}\frac{u_1}{N}\right)u_1\xi + \left(\frac{\partial N}{\partial u_2}\frac{u_2}{N}\right)u_2\xi + \ldots\ldots \quad \ldots\ldots 22$$

This expressed the relative change of N with respect to the sum of the relative changes of each variable. If the error or change that occurs in only one variable is considered, all the terms would go to zero, leaving

$$N\xi = \left(\frac{\partial N}{\partial u_1}\frac{u_1}{N}\right)u_1\xi \quad \text{...23}$$

The bracketed terms become dimensionless coefficient, which expresses the percentage of the relative variable change, transmitted to the relative dependent variable [11]. This is the sensitivity coefficient, which shows the relative importance of each of the variables to the model solution. Table 3 gives the development and solutions of the sensitivity equations for the spray mass flux, spray air velocity and spray deposition models developed respectively. Table 3 gives the independent variables and their respective sensitivity coefficients for all the models. Each of the sensitivity coefficients contains the values for all other variables considered in the model development; thus, the sensitivity of any one value is quite dependent upon the values for all other variables. A focus is achieved by determining the mean values of variables. The sensitivity coefficient for the spray mass flux model range from between-0, 05 and-0.67 for all the independent variables. All the possible % changes in u_i the corresponding changes that would occur in N is negligible. The sensitivity coefficient of the applied variables showed considerable variations. Each sensitivity coefficient is a solution of the complete bracketed term in the sensitivity equation using all the variable values. The coefficients for each variable have considerable variation and display constant sensitivity coefficients. However, both the high and low coefficients of all the inputs variables of the model developed indicate high degree of precision with which the treatment values were compared and is a good index of reliability of the experimental results. It also implies that the sensitivity of developed predicted model to change in the constituent independent variables at all percentages is negligible and not significant.

Table 5: sensitivity coefficients of spray mass flux model parameters.

Spray mass flux model	
Parameter	Sensitivity Coefficient
A	0.27
Q_l	0.67
Q_a	-0.5
X	0.48
ρ_a	0.21

4. CONCLUSION

In developing a mathematical prediction model, some important steps were considered for proper formulation. The steps consist of model formulation and verification. The prediction of spray mass flux was done considering the change in mass flux and velocity of the spray as it moves toward the canopy. The assumptions made in this modeling are as follows; 1) the sprayer's air stream is treated as a plane symmetric jet in this study because the sprayer has air outlets that generate fan jets which are amenable to plane jet treatment [9]. 2) the liquid mass evaporated from the spray is entirely absorbed by the air in the control volume. 3) in determining airflow rate, there is the absence of co-flow (no ambient air parallel to jet air), 4) the change in distance χ, ($\Delta\chi$) is so small that there is no change in air velocity while moving through the distance, and 5) the air jet adjusts during travel from the sprayer air outlet to the tree canopy.

Predicted model was developed using dimensional analysis to assess the effects of application parameters on spray mass flux. The model structure was based on air velocity, spray droplet size, canopy foliage and tree characteristics. Overall model prediction agreed well with experimental data.

All the inputs variables of the model developed indicate high degree of precision with which the treatment values were compared and are a good index of reliability of the experimental results. The sensitivity of developed predicted models to change in the constituent independent variables is negligible and not significant.

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